Homework #5, PHY 674, 04 October 1995

t G be a group of order n. The regular character is given by

$$\chi_{\text{reg}}(g) = \begin{cases} 0 & \text{if } g \in G,\ g \neq e \in G \text{ and} \\ n & \text{if } g = e \in G. \end{cases} \tag{21.1}$$

ılculate the regular character for the equilateral triangle group. Now calculate

$$\sum_{i=1}^{s} d_i \chi_i(g) \tag{21.2}$$

· all $g \in G$ and compare. Surprised ? (s is the number of classes and d_i the dimension of the i-th educible representation.) See the script for details (4 points).

t up the regular representation of the equilateral triangle group. That is to say: For each element g in ɘ group write down the 6×6 matrix that represents g. Here's how to do it (in case you don't like the finition in the script): Take your multiplication table for C_{3v} and rearrange it in such a way that each ːment faces its inverse, that is the all diagonal elements are the neutral element. Now, the matrix ɔresenting the group element g is obtained in this way: If the entry in the multiplication table is g, ite down 1 in the corresponding place in the matrix, otherwise write down 0. Now that you have all ɘ matrices of the representation, find the character of the regular representation. Does it agree with ıat you calculated in the previous problem ? Is the character really a class function, i.e., constant thin classes ? I will spare you the task of finding a 6×6 matrix that reduces this representation to ɔlock-diagonal form showing how the regular representation reduces into irreducible representation.

t R be a rotation by 90° about the x-axis. How do the following functions transform under this ːation ? (In other words, given R and f calculate $P_R f$ for the following functions.)

ı $f(\vec{r}) = x\phi(r)$,
ı $f(\vec{r}) = y\phi(r)$,
ı $f(\vec{r}) = z\phi(r)$,
ıere $r = \sqrt{x^2 + y^2 + z^2}$.

ow that

$$(T_{m\vec{v}} - 1)\, H(\vec{r}, \vec{p}, t) = \left[H, \vec{G}\right] \cdot \vec{v} \tag{24.3}$$

· all functions H and all (sufficiently small) vectors $\vec{v} = \dot{\vec{r}}$, where $\vec{G}$ is given by

$$\vec{G} = m\vec{r} - t\vec{p} \tag{24.4}$$

d $T_{m\vec{v}}$ is the operator acting on H like

$$T_{m\vec{v}} H(\vec{r}, \vec{p}, t) = H(\vec{r} - t\vec{v}, \vec{p} - m\vec{v}, t). \tag{24.5}$$

› NOT assume that $\vec{p} = m\vec{v} = m\dot{\vec{r}}$ for this problem.

lems are worth 4 points each.